Synthesizing Open Worlds with Constraints using Locally Annealed Reversible Jump MCMC – Supplemental Materials

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In this document, we give the formulations of the distributions used in the main text.

1 Formulating Soft Constraints

Most of the constraints used in the examples employ modulated Gaussian and sigmoid functions to express soft constraints over real-valued scoring functions (Figure 1). Here, we describe them in detail.

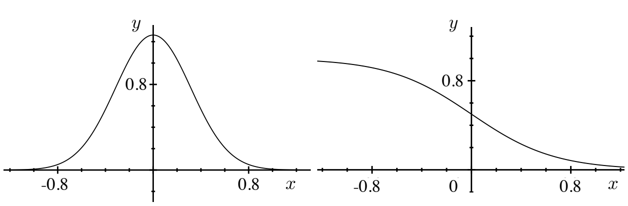
Let $\mathcal{N}(x, \mu, \sigma^2)$ be a Gaussian density with mean μ and variance σ^2 evaluated at x, and $\mathrm{Sig}(x; h) = \frac{1}{1+e^{-hx}}$ be a sigmoid function with h controlling the steepness. We formulate soft versions of logical predicates over real numbers as follows. All such values are computed in log space.

$$\begin{aligned}
\mathrm{Eq}(x, y, \sigma^2) &= \frac{\mathcal{N}(0, \|x-y\|, \sigma^2)}{\mathcal{N}(0, 0, \sigma^2)} \\
\mathrm{Greater}(x, y, h) &= \mathrm{Sig}(x-y; h) \\
\mathrm{Less}(x, y, h) &= \mathrm{Sig}(y-x; h) \\
\mathrm{Range}(x, y, z, h) &= \mathrm{Greater}(x, z, h)\mathrm{Less}(y, z, h).
\end{aligned}$$

The composition of several such functions yields a constraint whose values range from 0 (maximally unsatisfied) to 1 (maximally satisfied).

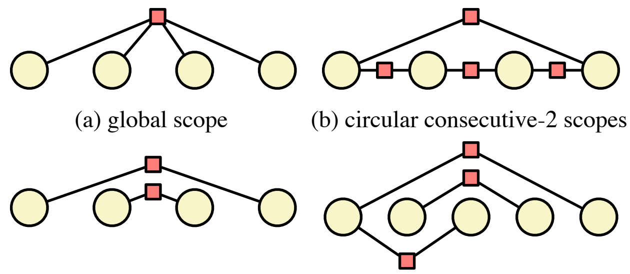
2 Simple String Example

Here, we give more details on the synthetic distributions over strings used in the paper to compare statistical efficiency of LARJ-MCMC versus other algorithms. We used two such synthetic distributions, both over strings $\mathbf{S} = S_1 S_2 \ldots S_N$ of different lengths consisting of random characters. It is feasible to calculate the normalization constant of these distributions analytically. Following are descriptions of the domains and factors of each distribution. Let N be the current number of letters in the string.



(a) $y = \mathcal{N}(x; 0, 0.1)$ (b) $y = \mathrm{Sig}(x; 3.0)$

Figure 1: *Gaussian and sigmoid functions used in designing soft constraints.*



(a) global scope (b) circular consecutive-2 scopes

(c) opposing scopes (even) (d) opposing scopes (odd)

Figure 2: *Visualization of the scopes being used in the simple string example.*

2.1 A distribution with global constraints

Domain. $\{$`'a'`,`'b'`$\}$. N can be $5 \sim 10$.

Scope and factor. There is one global factor whose scope is the entire string $\mathbf{S}$ (see Figure 2(a)). This factor constrains the string to be all `a`'s or all `b`'s depending on the length:

$$f(\mathbf{S}) = \begin{cases} Eq(\|\mathbf{S}\|, \|\{S_i | S_i = \texttt{'a'}\}\|, 0.2) & \text{if } \|\mathbf{S}\| \text{ is odd} \\ Eq(\|\mathbf{S}\|, \|\{S_i | S_i = \texttt{'b'}\}\|, 0.2) & \text{if } \|\mathbf{S}\| \text{ is even} \end{cases}$$

2.2 A distribution with local constraints

Domain. $\{$`'a'`,`'b'`,`'c'`$\}$. N can be $6 \sim 9$.

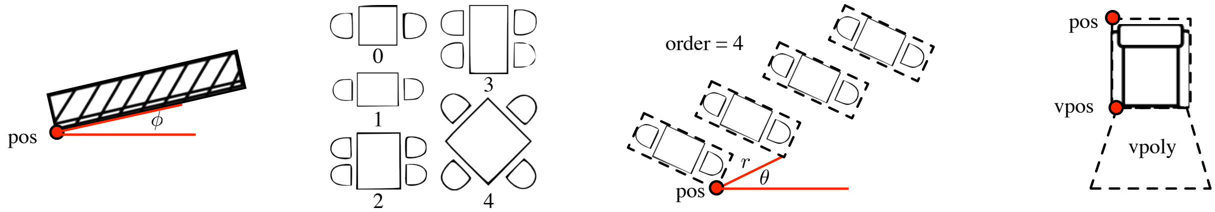
Scopes and factors.

- One factor is applied over each pair of circular consecutive characters $\{S_i, S_{i+1 \mod N}\}_{i=0}^{N-1}$. This scoping is illustrated in Figure 2(b). This factor constrains each such pair to be different, using the following factor function:

$$f(S_i, S_j) = \begin{cases} 0 & \text{if } S_i \neq S_j \\ \log(0.2) & \text{if } S_i = S_j \end{cases}$$

- Another factor is applied over pairs of opposing characters $\{S_i, S_{N-i-1}\}_{i=0}^{\lfloor N/2 \rfloor -1}$ (with an additional pair $\{S_0, S_{\lfloor N/2 \rfloor}\}$ if N is odd). This scoping is illustrated in Figure 2(c)-(d). The factor constrains all such pairs to be the same, using the following factor function:

$$f(S_i, S_j) = \begin{cases} 0 & \text{if } S_i = S_j \\ \log(0.05) & \text{if } S_i \neq S_j \end{cases}$$



(a) shelf (b) types of tables (c) table group (d) sofa

Figure 3: *Visualization of variables used in cafe layout example.*

3 Cafe Layout

3.1 Random variables

Each piece of furniture or a group of tables O contains the following attributes:

- O.pos $\in \mathbb{R}^2$ (Figure 3(a, c, d)).
- O.orientation $= \phi \in \{0, \frac{1}{12}\pi, \ldots, 2\pi\}$ for table groups, $\{0, \frac{1}{8}\pi, \ldots, 2\pi\}$ for sofas and shelves. (see Figure 3(a)).
- O.type $\in \{0, \ldots, 4\}$ for table groups, $\{0, 1\}$ for sofas. (see Figure 3(b)).

Each group of tables also includes the following attributes (Figure 3(c)):

- O.offset $= (r, \theta) \in \mathbb{R}^+ \times \{0, \frac{1}{4}\pi, \ldots, 2\pi\}$.
- O.order $\in \{2, \ldots, 10\}$.

Sofas contain these additional attributes (Figure 3(d)):

- O.vpoly to denote view area.
- O.vpos to denote position of view area.

Figure 3 shows how these parameters affect the layout visually.

3.2 Constraint specification

Figure 4 shows the factor functions used in the cafe layout example. The following is pseudocode for certain functions used in the constraints.

```
// bbox: gets the axis-aligned bounding box of an object.
// rotate_bbox: rotates the points of a bbox about the
     origin by the given angle.
Spacing(O) {
    b = bbox(rotate_bbox(bbox(O), O.orientation));
    r, theta = O.offset;
    spacing = max(abs(b.width - r * cos(theta)), abs(b.
        height - r * sin(theta)))
    return spacing;
}
```

4 Golf Course Layout

4.1 Random variables

Each course is parameterized by the following entities (see Figure 5):

- The path of the course is a list of P_i (two points per hole; 18 points in a 9 hole course)
- Each hole contains a start, middle, and end control point. The start and end of each hole correspond to a pair of path points P_i.
- Greens and fairways are defined by blob control point objects, each of which has a radius and position attribute.
- Flags F_i contain a single position attribute.
- Traps T_i are a collection of blob control points, all sharing a common radius.

4.2 Constraint specification

Figure 6 contains a list of factor functions used in the golf course layout example. The following is pseudocode for the functions used. `Straight(p1, p2, p3)` computes the magnitude of the mean curvature for a discrete curve segment [Sullivan 2006].

```
OutsideBlob(pos, blob, size_param){
  return Less(0.5, ImplicitShapeFn(pos, blob.
       control_points, size_param), 100);
}

InsideBlob(pos, blob, size_param){
  return Greater(0.5, ImplicitShapeFn(pos, blob.
       control_points, size_param), 100);
}

ImplicitShapeFn(pos, control_points, size_param){
  val = 1.0;
  for(cp in control_points){
    dist = d(pos, cp.pos);
    val -= (4/9)*(dist/( cp.rad + size_param))^6 - (17/9)
        *(dist/(cp.rad + size_param))^4 + (22/9)*(dist/(
        cp.rad + size_param))^2;
  }
  return val;
}

AvoidIntersection(s1, s2, line_width){
  a1 = s1.first; a2 = s1.second;
  b1 = s2.first; b2 = s2.second;
  if(Intersecting(a1, a2, b1, b2)){
    cross = IntersectionPoint(a1, a2, b1, b2);
    amid = Midpoint(a1, a2);
    bmid = Midpoint(b1, b2);
    dist_to_midpts = d(cross, amid) + d(cross, bmid);
    max_dist_to_midpts = d(a1, amid) + d(b1, bmid);
    intersect_factor = 2 - dist_to_midpts /
        max_dist_to_midpts; // more intersection if
        closer to midpoint
    Greater(intersect_factor * 2 * line_width, 0, 1.0);
  }
  else{ // mutual minimum distance from pt to line
    dist = min(DistPtLine(a1, b1, b2), DistPtLine(a2, b1,
        b2),
             DistPtLine(b1, a1, a2), DistPtLine(b2, a1, a2)
                 );
    Greater(2 * line_width, dist, 1.0);
  }
}
```

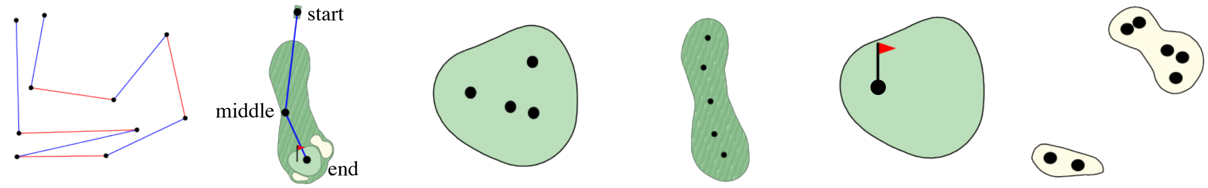
```
Straight(p1, p2, p3) {
   return discrete_curvature(p1, p2, p3);
}
```

References

SULLIVAN, J. M. 2006. Curvature measures for discrete surfaces. In *ACM SIGGRAPH 2006 Courses*, ACM, New York, NY, USA, SIGGRAPH '06, 10–13.

Description	Scope	Factor Function	Weight
non-overlap	each object pair O_i, O_j	$Eq\left(0, \frac{A(P(O_i) \cap P(O_j))}{\min\{A(P(O_i)), A(P(O_j))\}}, 0.1\right)$	40
inside room R	each object O_i	$Eq\left(0, \frac{A(P(O_i) - P(R))}{A(P(O_i))}, 0.1\right)$	40
align to nearest segment W	each table group, shelf, and plant O_i	$Eq\left(0, \sin(\Theta(O_i, W)), 0.1\right)$	40
spacing within group	each table group O_i	$Eq\left(80, \text{Spacing}(O_i), 400\right)$	30
distance to nearest segment W	each shelf O_i	$Eq\left(0, d_s(O_i.\text{pos}, W), 10^4\right)$	40
occupied area in zone Z	all sofas $\mathbf{O}$	$Eq\left(0.6, \frac{A((\cup P(O_i)) \cap P(Z))}{A(P(Z))}, 0.2\right)$	30
occupied area in Room R	all table groups $\mathbf{O}$	$Eq\left(0.6, \frac{A((\cup P(O_i)) \cap P(R))}{A(P(R))}, 0.1\right)$	30
total number of different types	all table groups $\mathbf{O}$	$Eq(2, \|\{O_i.type \| O_i \in \mathbf{O}\}\|, 1)$	30
balance w.r.t. point c	all table groups $\mathbf{O}$	$Eq(0, d(\frac{\sum A(P(O_i))O_i.\text{pos}}{\sum A(P(O_i))}, c), 10^5)$	30
encourage conversation	each sofa pair O_i, O_j	If $d(O_i, O_j) < 500$, then $Eq\left(1, \frac{A(O_i.\text{vpoly} \cap O_j.\text{vpoly})}{A(O_i.\text{vpoly})}, 0.1\right)$	1
closeness 1	each sofa pair O_{2i}, O_{2i+1}	$Range\left(100, 500, d(O_{2i}.\text{pos}, O_{2i+1}.\text{pos})\right)$	1
closeness 2	each sofa pair O_{2i}, O_{2i+1}	$Eq\left(0, d(O_{2i}.\text{vpos}, O_{2i+1}.\text{vpos}), 200\right)$	1
inside sofa zone Z	each sofa O_i	$\max\{Less\left(0, d_{poly}(O_i.\text{pos}, Z)\right), Less\left(0, d_{poly}(O_i.\text{endpos}, Z)\right)\}$	1
close to sofas $\mathbf{S}$	all lamps $\mathbf{O}$	$Less\left(75, \min_{S_i \in \mathbf{S}} d_{poly}(O_i.\text{pos}, P(S_i)), 5\right)$	1
cover all tables, sofas, shelves $\mathbf{S}$	all plants $\mathbf{O}$	$\sum_{S_i \in \mathbf{S}} Range\left(-1.0, \max_{O_i \in \mathbf{O}} d_{poly}(O_i.\text{pos}, P(S_i)), 4\right)$	1
cove all sofas $\mathbf{S}$	all lamps $\mathbf{O}$	$\sum_{S_i \in \mathbf{S}} Range\left(-1.0, \max_{O_i \in \mathbf{O}} d_{poly}(O_i.\text{pos}, P(S_i)), 4\right)$	1

Figure 4: *Factor specification for cafe layout examples. $P(O)$ returns the bounding polygon associated with object O, $\Theta(X, Y)$ returns the difference in angle between line segment Y (relative to a global coordinate frame) and the orientation attribute of object X. $A(x)$ returns the area of polygon x, and $\cup, \cap$ denote union/intersection operations on polygons. $d(x, y)$ returns the distance from point x to point y. $d_s(x, s)$ returns the distance from point x to line segment s. $d_{poly}(x, p)$ returns the squared distance from a point x to a polygon p, multiplied by -1 if the point is inside. $Spacing(O)$ denotes the internal spacing in a table group O, given in the pseudocode. The result of each factor function is multiplied in log space by its corresponding weight.*



(a) path (b) hole (c) green (d) fairway (e) flag (f) sand traps

Figure 5: *Visualization of variables used in the golf course layout example.*

Description	Scope	Factor Function	Weight
inside course C	each control point P_i of path	$Greater\,(30, DistPoly(P_i, C))$	1
outside lakes L	each flag and each control point P_i of path, green, and sandtrap	$OutsideBlob\,(P_i, L, 30)$	1
distance to target location t_i	each path end point P_i	$Less\,(20, d(t_i, P_i), 5)$	1
non-overlap	path segment pairs S_i and S_j	$AvoidIntersection\,(S_i, S_j, 40)$	5
par distribution for par 3, 4, 5	for all path segments $\mathbf{S}$. t_n is the target number of paths with par n	$\sum_{n=3}^{5} Eq\,(t_n, \|\{s_i \in \mathbf{S} \| par(s_i) = n\}\|, v_n)$	1
total par count	all path segments $\mathbf{S}$	$Eq\left(36, \sum_{s_i \in \mathbf{S}} par(s_i), 5\right)$	1
length range $d_{i,min}, d_{i,max}$	each path segment S_i	$Range\,(d_{min}, d_{max}, L(S_i), 1)$	0.25
no sharp bends	each consecutive 3 path control points P_i, P_j, P_k	$Less(1.2, Straight(P_i, P_j, P_k), 100)$	1
hole straightness	each hole H_i	$Eq(0, Straight(H_i.\text{start}, H_i.\text{mid}, H_i.\text{end}), 0.2)$	1
target size range $r_{i_{min}}, r_{i_{max}}$	each control point radius R_i of green and fairway	$Range(r_{i,min}, r_{i,max}, R_i, 1)$	1
distance to hole H_i	each control point P_i of green	$Less\,(15, d(H_i.\text{end}, P_i), 1)$	1
inside green G_i	each flag F_i	$InsideBlob\,(F_i, G_i, 30)$	1
fairway target location t_i	each fairway end point P_i	$Eq\,(0, d(t_i, P_i), 15)$	1
distance to hole path H_j	each fairway control point P_i	$Eq\,(0, d_{path}(P_i, H_j), 50)$	1
distance to hole path H_j	each sand trap control point P_i	$Range\,(17, 37, d_{path}(P_i, H_j), 15)$	1
outside green G_i	each control point of sand trap P_i	$OusideBlob\,(P_i, G_i, 60)$	1
sand trap location t_i	each sand trap control point P_i	$Eq\,(50, d(t_i, P_i), 2500)$	1
distance between traps	each sand trap pairs T_i, T_j	$Greater\,(50, d(C(T_i), C(T_j)), 1)$	1

Figure 6: *Factor functions used in the golf course layout example.* $d(x, y)$ *denotes distance between two points.* $d_{path}(x, y)$ *denotes the distance from a point to a path.* $C(x)$ *denotes the centroid of the control points of object* x.